

June 2005 Article of the Month

By Rod Machado

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Why V_x and V_y Change With Altitude?

Dear Mr. Machado

I have been flying for a while but have started to read (and enjoy a great deal) your work only relatively recently. I was particularly impressed by the clarity of your recent explanation of maneuvering speed V_a , which has prompted me to suggest another subject: During flight training we have all been taught that V_x increases and V_y decreases with altitude, converging at either the "service ceiling" or "absolute ceiling" of the aircraft--I forget which. However, in twelve years of flying I have never received a clear explanation of why this is the case. One CFI, to his credit, did some research and then passed on to me a quite technical explanation. However, since I lacked what appeared to be the requisite Ph.D. in fluid mechanics, my understanding of the subject was, shall we say, only modestly enhanced.

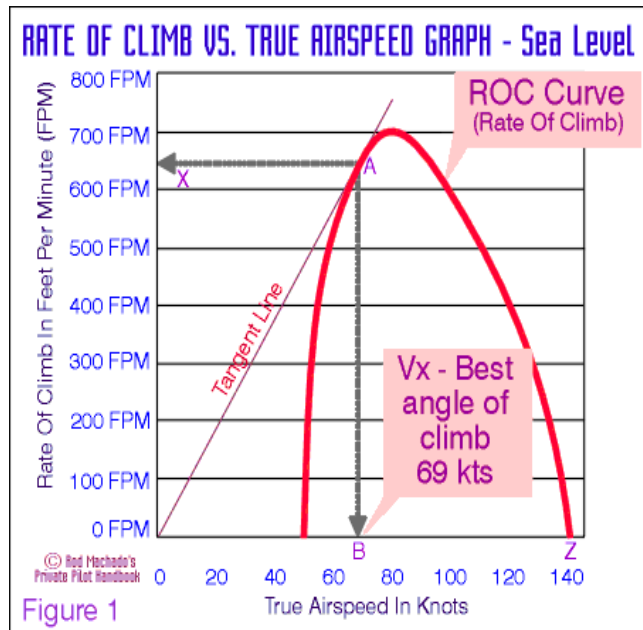
Thanks,
K.M.

Greetings Mr. K:

Thanks for those wonderful comments. Your question about V_x and V_y is a good one. In fact, it's one of the more difficult things to explain. Any explanation requires the association of several ideas to adequately answer the question. Here's my attempt.

Let's first start with why V_x increases with altitude.

As you know, V_x is the best angle of climb speed. At this speed the airplane acquires the most altitude (vertical movement) for a given distance (horizontal movement) over the ground. Take a look at Figure 1 below.

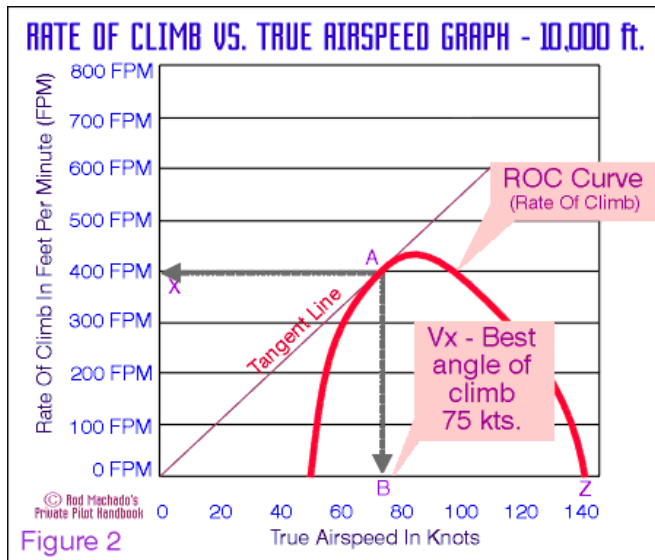


The red line is the rate of climb curve. Let's say I placed you in the far left hand corner of the reference box at the 0-FPM and 0-airspeed corner. If I asked you to look up from that position and point to the top of the curve, your arm would point in the same direction as the tangent line shown in the figure. The angle your arm makes with the horizontal represents the maximum angle you can point upward and still touch the rate of climb curve (ROC) at point A. Thus, mathematically speaking, the maximum upward angle relative to the horizontal, represents the maximum "angle" of climb for your airplane¹. If you drop straight down from point A, you'll find point B, which represents an airspeed of 69 knots. Therefore, our best angle of climb speed at sea level is 69 knots.

Now, ask yourself what happens to the ROC curve when altitude increases. Since power decreases with altitude, the ROC curve will change shape and grow shorter as shown in Figure 2. Naturally, this is the result of the airplane's ROC decreasing at higher altitudes.

¹ Note from Ali: When the tangent to the curve goes through the origin, then small changes in airspeed will have exactly proportionate changes in rate of climb and no change in angle of climb. The airspeed where this occurs is referred to as V_x .

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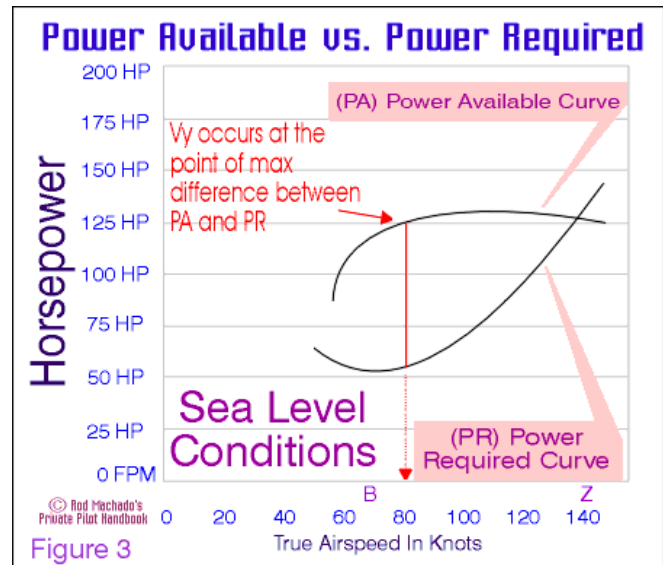


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If you apply the same logic, it stands to reason that a shorter ROC curve causes the tangent line to make less of an angle with the horizontal. You can think of this as an airplane climbing at a geometrically smaller climb angle relative to the horizontal as altitude increases. Therefore, dropping straight down from the tangent point (point A) to point B, we find that the best angle of climb speed has increased to 75 knots. And this is the basic reason why V_x increases with altitude. Describing it this way sure beats trying to explain the same thing using differential calculus.

Let's move on to why V_y changes with altitude.

V_y is the best rate of climb speed. It's the speed that results in the greatest upward deflection on the VSI. Since an airplane's rate of climb is a function of excess power, it stands to reason that the more excess power you have the higher the rate of climb will be. Take a look at Figure 3 below.

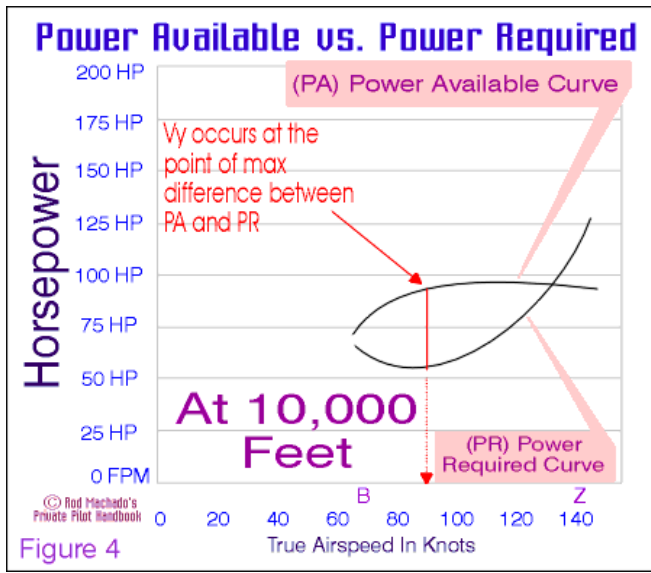


The power required curve (PR) shows the power required to maintain level flight. It's curved because of the effects drag. Remember, induced drag increases as the airplane slows and decreases as it speeds up. Parasite drag, on the other hand, decreases as the airplane slows and increases as it speeds up. The power available (PA) curve is also curved because propeller efficiency changes at different airspeeds. After all, the propeller is nothing more than a moving wing isn't it? Therefore, it experiences its own drag effects which are similar to the drag experience by the airplane's wings.

There is a point between the PA and PR curves where maximum separation exists (the vertical red line). This is the point where you have the minimum power required for level flight and the maximum amount of excess horsepower capable of being produced by the engine. Following this point down to the bottom of the graph tells you the airplane's best rate of climb speed. In this example, V_y is approximately 82 knots true airspeed (TAS).

Now ask yourself what happens to the PA and PR curves as altitude increases. Yes, they also change shape and position on the graph as shown in Figure 4.

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Notice how the curves came closer together and moved slightly to the right. This should make sense since the power available (PA) curve must move closer to the bottom of the graph with an increase in altitude (we're assuming the airplane isn't turbocharged). The power required (PR) curve moves up and shifts to the right slightly. Find the point of maximum separation between the two newly shifted curves (the vertical red line). Move straight down to the airspeed line and you'll see that V_y has actually *increased* with an increase in altitude (it's now 90 knots, TAS). Now hold on for a second because I realize that this appears contradictory to what you've heard about V_y . In reality, it isn't contradictory. Let me explain why with two basic observations.

OBSERVATION #1

If V_y is measured as a true airspeed, it increases *approximately* 1% for every thousand feet increase in altitude. That's right! If you measure V_y in terms of TAS it will increase, not decrease with altitude. Therefore, if V_y is 82 knots (TAS) at sea level, it must be approximately 90 knots (TAS) at 10,000 feet. (Here's the math: 10% of 82 is 8.2, so add 8.2 to 82 and you get an approximate value of 90 knots.)

Now ask yourself what IAS is necessary to give you a TAS of 90 knots at 10,000 feet? How do you figure this out? You work backwards from the old rule of thumb you were taught.

OBSERVATION #2

Weren't you taught that TAS increases 2% per thousand feet for any given IAS? Let's assume for a second that our IAS at 10,000 feet is 75 knots (I picked this value for a good reason as you'll soon see). What's our TAS? It must be 90 knots. (Here's the math: At 10,000 feet, our TAS has increased by 20% above an IAS of 75 knots. Take 20% of 75, which is 15. Add this to 75. This equals 90.)

Therefore, if our TAS at 10,000 feet is 90 knots, then our IAS is 75 knots. You're almost done. Ask yourself one last question. What's the value of V_y in IAS at sea level? Aren't IAS and TAS the same at sea level? Assuming no airspeed indicator error, they are indeed equal. Therefore, according to Figure 3, V_y , as an IAS, is 82 knots at sea level.

And here's the final result. At sea level, V_y is 82 knots IAS. At 10,000 feet V_y is 75 knots IAS. So V_y , as an IAS, has actually decreased with an increase in altitude.

That's it. You're finished with the problem. Pretty easy right?

Even though these Figures don't represent a specific airplane you can see that at 10,000 feet V_x equals V_y (both are 75 knots). Therefore, we can say that, for this fictitious airplane, 10,000 feet is the airplane's absolute ceiling.

Whew! That's a lot. If you stick with it you'll eventually understand the explanation. It's as simple as I can make it without having to disregard the science behind the effect. I hope this helps.

Best,
Rod